The logic of words:
A monadic decomposition of lexical meaning

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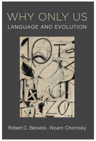
Overview

Overview

"The logic of words"? Where?!

Words are the unique building blocks of human language.



The minimal meaning-bearing elements of human languages . . . are radically different from anything known in animal communication systems. Their origin is entirely obscure, posing a very serious problem for the evolution of human cognitive capacities, language in particular. (Berwick & Chomsky 2016:90)

A most noticeable characteristic of words is their arbitrariness.

The linguistic sign is arbitrary. The idea of "sister" is not linked by any inner relationship to the succession of sounds s-ö-r which serves as its signifier in French; that it could be represented equally by just any other sequence is proved by differences among languages and by the very existence of different languages. (Saussure 2011:67–68)

Isn't logic and arbitrariness an antinomy?🤔

Lexical vs. compositional semantics

Research on word meanings and that on sentence meanings have been kept separate in linguistics. Formal logical tools are applied to the latter.

- Lexical semantics: the study of word meanings (see Cruse 1986)
- Compositional semantics: the study of how meanings of larger expressions are formed from meanings of their smaller parts (see Aloni & Dekker 2016)

Topics in lexical & compositional semantics

- Ambiguity and polysemy
- Synonymy, antonymy, etc.
- Lexical type inheritance
- Semantic roles
- etc.

- Extension vs. intension
- Quantification
- Presupposition
- Modality
- etc.

The role of words in compositional semantics

Arbitrary meanings are generally treated as constants of the model (i.e., taken for granted). What matters is their semantic types in composition.

(1) a. $[\![\text{smokes}]\!] = \lambda x \in D_e \,.\, x$ smokes $\quad e \to t$

b. $[\![\text{loves}]\!] = \lambda x \in D_e \,.\, [\lambda y \in D_e \,.\, y$ loves $x] \quad e \to e \to t$

c. $[\![\text{Ann}]\!] =$ Ann $\quad e \quad$ (Heim & Kratzer 1998:44)

(In H&K's system, t is the truth value type, e is the individual type, and D_e is the domain of individuals.)

The λ-terms clarify the semantic types of words, but the real arbitrary parts remain intact (as convenient labels). They are deemed largely irrelevant to compositional semantics.

Lexical decomposition

Some linguistic theories decompose lexical words into abstract, subatomic units, such as

1. Generative semantics (see Lakoff 1971; cf. Dowty 1979)
2. Lexical conceptual structure (Jackendoff 1972)
3. Lexical relational structure (Hale & Keyser 1993; cf. Levin & Rappaport-Hovav 1995)
4. (neo-)Davidsonian event semantics (Davidson 1967, Parsons 1990; cf. Pustejovsky 1996)
5. Distributed morphology (Halle & Marantz 1993, 1994, Marantz 1997)
6. Exoskeletal syntax (Borer 2005ab, 2013)

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✏ The last three are currently still popular. ④ is about semantics, while ⑤⑥ (collectively known as "root syntax") are more about morphosyntax.

Lexical decomposition in semantics

Examples:

(2) a. kill := $(\text{CAUSE } (x, (\text{BECOME } (\text{NOT}(\text{alive } x)))))$
(generative semantics)

b. John flew the plane from New York to Boston :=
$\text{CAUSE}(\text{John}, \text{GO}(\text{plane}, [_{[\text{path}]}\ \text{FROM } (\text{IN } [\text{Boston}]), \text{TO}(\text{IN } [\text{NewY}])])))$
(lexical conceptual structure) (Pustejovsky 2016:53–54)

c. Jones buttered the toast :=
$\exists e[\text{butter}(e) \wedge \text{AGENT}(e) = \text{Jones } \wedge \text{THEME}(e) = \text{toast }]$
(neo-Davidsonian event semantics) (Landmann 2000:1–2)

The uppercase labels denote reserved keywords (i.e., special predicates).

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The uppercase labels denote reserved keywords (i.e., special predicates).

✏ However, the arbitrary core meanings of words are left as such.

Lexical decomposition in syntax

Root syntax: a popular trend in current Chomskyan linguistics

- Halle & Marantz (1993 et seq.): Distributed Morphology (DM)
- Borer (2005, 2013): Exoskeletal Syntax (XS)
- Chomsky (2019):

morphology IS syntax

> *If you accept—as I am doing here—the Hagit Borer–Alec Marantz theory of root categorization, which I think is pretty strongly motivated, the roots in the lexicon are independent of category.*

A theory-neutral definition of “root”

A root is a purely lexical unit in syntactic derivation that is void of categorial information. It only acquires a syntactic category (and thereby a categorized interpretation) by externally combining with one.

Example (DM):

N → ⟨/dɔg/, ‘🐶’⟩

n $\sqrt{\text{DOG}}$ →⟨?, ?⟩

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n $\sqrt{314}$ →⟨?, ?⟩ (Harley 2014)

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Example: root-and-pattern morphology in Hebrew (Arad 2005:16)

$\sqrt{}$Š-M-N 'having to do with some fatty substance' ⇛

šamen 'adj. fat', *šemen* 'n. oil', *šamenet* 'n. cream', *šuman* 'n. fat', *šaman* 'v. grow fat', *šimen* 'v. grease', *hišmin* 'v. fatten'

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Example: root-and-pattern morphology in Hebrew (Arad 2005:16)

$\sqrt{}$X-Š-B 'having to do with some mental activity or mental state' ⇛

xašav 'v. think', *xišev* 'v. calculate', *hexšiv* 'v. consider', *maxšev* 'n. computer', *maxšava* 'n. thought', *taxšiv* 'n. calculus', *xešbon* 'n. account'

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> *I consider roots to be underspecified lexical cores. These cores are potentialities, which may be incarnated in many different ways. (Arad 2005:12)*

Some linguistic motivations for Root Syntax

(see Siddiqi 2009, Borer 2013, Bobaljik 2019, and Embick 2020 among many others for more info)

1. The Single Engine Hypothesis (i.e., Occam's Razor)
   A grammatical theory with one generative engine (i.e., syntax) is more parsimonious than a theory with two engines (i.e., syntax + lexicon).
2. The pretheoretical, prescientific nature of the notion "word"
   "Word" is notoriously difficult to define due to the mingling of multiple perspectives (phonological, orthographic, etc.). Linguists need an atomic syntactic object, which is not necessarily a "word" in the everyday sense.
3. The root-category separation is empirically useful; e.g.,
   – nominalization: *his quickly marrying her* vs. *his quick marriage to her*
   – zero derivation: $dog_{N/V}$, $run_{N/V}$, $board_{N/V}$
   – idiomaticity: *cat*/*dog*/*table* vs. *editor*/*natural*/*classify*

NB it's not that any single reason categorically motivates Root Syntax, but that many independent desiderata are attained in a coherent, unified way if linguists push syntactic analysis to the root level.

A syntax-semantics mismatch

Observation: Lexical decomposition is practiced in both semantics and syntax in theoretical linguistics, but with unbalanced development.

Bare words are decomposed in syntax but not in semantics.

Compare neo-Davidsonian event semantics & root syntax:

(4) a. Jones buttered the toast.

b. $[\![\text{butter}]\!] = \lambda e\,.\,\text{BUTTER}(e)$ (here e is the event variable)

c. $[_{\text{V}}\ v\ \sqrt{\text{BUTTER}}\,]$ $\rightarrow \langle$/ˈbʌtɚ/, $\lambda e\,.\,\text{BUTTER}(e)\rangle$

Semantically the bare verb *butter* is treated as a terminal node denoting a typed function, whereas syntactically it is nonterminal and has a further layer of structure (i.e., the root structure).

This study

Problem detected

A part of syntactic structure is systematically left out in semantic interpretation.

Goal

Solve the above syntax-semantics mismatch by logical tools.

Tool

The category-theoretic “monad” (the writer monad from functional programming)

Idea

Separate the logical and the arbitrary part of word meaning in a systematic way.

Key references

Asudeh & Giorgolo (2020), Song (2021b)

Overview

Source of inspiration

Asudeh & Giorgolo (2020) use monads to compose Potts's (2005, 2007) "at-issue" (truth-conditional) and "side-issue" (non-truth-conditional) meanings. I am inspired by their treatment of conventional implicature.

(5) a. Donald is a **Yank**.

b. This **cur** bit me. (Asudeh & Giorgolo 2020:13)

Words like *Yank* and *cur* carry speaker attitudes besides their basic meanings. A&G view these as conventional, non-truth-conditional.

The arbitrary core of word meaning is a matter of conventionalization too. Moreover, the meaning space of roots in Root Syntax is by definition not compositional but "encyclopedic"—like a warehouse of all sorts of arbitrary semantic/pragmatic information associated with linguistic forms.

The writer monad

Monad is a general concept from category theory, but the writer monad A&G use for conventional implicature is from functional programming.

> *[T]he writer monad [is] used for logging or tracing the execution of functions. It's also an example of a more general mechanism for embedding [side] effects in pure computations. (Milewski 2019:49)*

(6) **type** Writer a = (a, **String**)
(*Writer a* creates a log area for any type *a*)

(7) a. Yank :: (American, {The speaker has a negative attitude.})

b. Donald is a Yank :: (American(Donald), {negative attitude})

NB *American* in (7) is a predicate label, which shows that A&G do not decompose bare words (unsurprising!). But since the monadic tool is highly general, we can easily extend their theory to our problem at hand.

Proposal: Root Syntax is monadic

An important advantage of Root Syntax is that it completely separates the formal-logical and the idiosyncratic part in the syntactic analysis of natural languages. The latter is deemed irrelevant to syntactic computation.

> *[I]t is not clear that the computational system of language . . . must know whether a node contains "dog" or "cat." . . . [T]his difference . . . is a matter of Encyclopedic knowledge. . . . (Marantz 1995:4)*

By analogy, syntactic computation is "pure," while the arbitrary meanings that are introduced into the computation via words and carried along throughout it can be considered "side effects."

✏ In fact, Root Syntax matches the monadic computational pattern even more closely than A&G's conventional implicature (since the latter still has "nonpure" stuff in its pure function slot).

A bit more linguistics

Perhaps it is best to keep arbitrary word meanings out of pure, model-theoretic computation anyway, for independent reasons:

> *[A] lexical item provides us with a certain range of perspectives for viewing what we take to be the things in the world, or what we conceive in other ways; these items are like filters or lenses, providing ways of looking at things and thinking about the products of our minds. The terms themselves do not refer . . . . (Chomsky 2000:36)*

> *[A]n extensionalist semantic approach, where basic terms of the semantic representation are ultimately defined by what they are true of, in one or more than one world . . . cannot possibly shed much light on those aspects of lexical semantic competence based on oppositions in conceptualization rather than in distinct extensions: [e.g., home vs. house, broad vs. wide, use vs. utilize], to say nothing about notorious problematic cases like [time, air, or god]. (Acquaviva 2014:281)*

The monadic tool offers linguists a potential way to reconcile the tension between model-theoretic semantics and anti-referentialism.

Definitions (A&G)

In category theory, a monad is an endofunctor together with two natural transformations, respectively called *unit* (η) and *multiplication* (μ). In functional programming, an additional operator *bind* (>>=) is usually defined based on μ. A&G further defines an ancillary operation *write*.

- $\eta(x) = \langle x, e\rangle : a \to \text{Writer } a$ *embeds a value in a trivial wrapper (e is the empty string)*
- $\mu\langle\langle x, s_1\rangle, s_2\rangle = \langle x, s_1 \mathbin{++} s_2\rangle : \text{Writer } (\text{Writer } a) \to \text{Writer } a$
  combines log entries by concatenation (s_1 and s_2 are strings)
- $\langle x, s_1\rangle \mathbin{>\!>=} \lambda u.\langle f(u), s_2\rangle = \mu((\lambda\langle u, s\rangle.\langle\langle f(u), s_2\rangle, s\rangle)\langle x, s_1\rangle)$
  $= \mu\langle\langle f(x), s_2\rangle, s_1\rangle = \langle f(x), s_2 \mathbin{++} s_1\rangle$
  $: \text{Writer } a \to (a \to \text{Writer } b) \to \text{Writer } b$ *pure function and logging proceed in parallel*
- $\text{write}(s) = \langle 1, s\rangle$ *wraps a string in a dummy monadic term*

(8) a. $[\![\text{Yank}]\!] = \text{write}(p) \mathbin{>\!>=} \lambda y.\eta(\text{American}) = \langle \text{American}, \{p\}\rangle$

b. $[\![\text{Yank}]\!] \mathbin{>\!>=} \lambda x.\eta([\![\text{a}]\!]([\![\text{is}]\!](x))([\![\text{Donald}]\!])) = [\![\text{Yank}]\!] \mathbin{>\!>=} \lambda x.\eta(x(\text{Donald}))$
$= \langle \text{American}, \{p\}\rangle \mathbin{>\!>=} \lambda x.\ \langle x(\text{Donald}), \emptyset\rangle = \langle \text{American}(\text{Donald}), \emptyset \cup \{p\}\rangle$
(p = The speaker has a negative attitude toward Americans.)

Definitions (new)

In the case of Root Syntax, we need to record which root tags which linguistic category, so the log area cannot simply be a set of propositions, but should be a set of ordered pairs instead.

Recall the root categorization schema $[_X \text{ X } \surd\,]$ (which categorizes a root into linguistic category X). I assign it the denotation

$$\text{write}(\langle \text{X}, \surd \rangle) \text{ >>= } \lambda y.\eta(⟦\text{X}⟧)$$

which writes $\langle \text{X}, \surd \rangle$ into the log slot of a vacuous monadic term.

(9) a. $⟦[_N\ n\ \surd\text{DOG}\ ]⟧ = \text{write}(\langle n, \surd\text{D}\rangle) \text{ >>= } \lambda y.\eta⟦n⟧ = \langle ⟦n⟧, \{\langle n, \surd\text{D}\rangle\}\rangle$
(an entity that is idiosyncratically characterized by $\surd$DOG)

b. $⟦[_V\ v\ \surd\text{BUTTER}\ ]⟧ = \text{write}(\langle v, \surd\text{B}\rangle) \text{ >>= } \lambda y.\eta⟦v⟧ = \langle ⟦v⟧, \{\langle v, \surd\text{B}\rangle\}\rangle$
(an event that is idiosyncratically characterized by $\surd$BUTTER)

Interim summary: Root Syntax 🤝 monadic semantics

Fundamental advantages

- The hole in syntax-to-semantics mapping concerning bare words is filled.
- Lexical decomposition is now balanced between syntax and semantics.
- We are one step closer to a reconciliation between model-theoretic semantics and linguistic anti-referentialism.

A consequence: Once monadic, always monadic.

Since words are the backbone of human language, the current theory entails that monadic composition is as important as function application in natural language semantics—it is the norm rather than the exception.

Bonus

The theory can be naturally extended from lexical words to **semilexical words.**

Overview

Semilexical (aka semifunctional) words

Not all words in natural languages are purely lexical or functional. There are also many in-betweens (see Song 2021a for a typological survey).

(10) a. *La pasta* ***va*** / ***viene*** *mangiata subito.* [Italian]
the pasta $\text{PASS}_{\text{obligatory}}$ $\text{PASS}_{\text{regular}}$ eaten immediately
'Pasta must be / is eaten immediately.' (Cardinaletti & Giusti 2001:392)

b. *yī* ***wèi*** / ***míng*** / ***gè*** *lǎoshī* [Mandarin]
one $\text{CLF}_{\text{respectful}}$ $\text{CLF}_{\text{professional}}$ $\text{CLF}_{\text{neutral}}$ teacher
'a teacher' (Song 2019:125)

c. *Em/Tao* ***không*** / ***đéo*** *cần anh/mày giúp.* [Vietnamese]
1SG.N/V $\text{NEG}_{\text{neutral}}$ $\text{NEG}_{\text{vulgar}}$ need 2SG.N/V help
'I do not need your help.' (Li Nguyen, p.c.)

☞ Hallmark: grammatical function + lexical coloration

logical-compositional ⇑ category

conventional-idiosyncratic ⇑ root

➥encyclopedic content, speaker attitude, register conditioning, etc.

Semilexical affixes

Not only words but also grammatical affixes can be semilexical.

(11) a. English derivational affixes:
-ity, *-al*, *-ic*, *-ness*. . . (same category, idiosyncratic usage)

b. Innu (Algonquian) verbal classifiers:
-eci- 'CLF.sheetlike', *-ašku-* 'CLF.long&rigid'. . . (Aikhenvald 2017)

c. West Greenlandic verbalizers:
-u 'V.be', *-nngur* 'V.become', *-gar* 'V.have', *-ssaaliqi* 'V.lack'. . .
(Fortescue 1984)

Semilexical (aka semifunctional/semigrammatical) items are prevalent in human languages yet have seldom been studied from a formal linguistic (esp. formal semantic) perspective. They fit in our theory quite naturally.

Generalized Root Syntax

Core features of classical Root Syntax:

- ultimate lexical decomposition
- complete separation of grammatical and idiosyncratic information
- super fine-grained, “subatomic” analysis

Limitation: confined to the lexical domain (basically a morphological tool)

Generalization (Song 2019)

Root-oriented thinking can be extended into the grammatical domain in a principled way and thereby be made into a more general analytical tool.

- Core motivation: lexical idiosyncrasy in the grammatical domain
- Similar ideas occur in Borer (2013), Acedo-Matellán & Real-Puigdollers (2019), and Pots (2020), but Song’s (2019) theory is a systematic extension of the Distributed Morphology incarnation of Root Syntax.

Monadic composition of semilexical items

Proposal: Semilexical items have monadic semantic types of the form

$$\langle \text{Pure Function}, \text{Root Log} \rangle$$

where the root log is the same one as that for lexical words. It systematically records which linguistic category (lexical or functional) is "colored" by what root.

(12) a. Mandarin classifiers: $[\![[_{\text{Cl}}\ \text{Cl}\ \sqrt{\text{WÈI}}\ ]]\!] =$
write$(\langle \text{Cl}, \sqrt{\text{WÈI}} \rangle)$ >>= $\lambda y.\, \eta [\![\text{Cl}]\!] = \langle [\![\text{Cl}]\!], \{\langle \text{Cl}, \sqrt{\text{WÈI}} \rangle\} \rangle$
(a classifier colored by $\sqrt{\text{WÈI}}$ [i.e., for people, respectful])

b. Vietnamese negation particles: $[\![[_{\text{Neg}}\ \text{Neg}\ \sqrt{\text{ĐÉO}}\ ]]\!] =$
write$(\langle \text{Neg}, \sqrt{\text{ĐÉO}} \rangle)$ >>= $\lambda y.\, \eta [\![\text{Neg}]\!] = \langle [\![\text{Neg}]\!], \{\langle \text{Neg}, \sqrt{\text{ĐÉO}} \rangle\} \rangle$
(a negator colored by $\sqrt{\text{ĐÉO}}$ [i.e., vulgar])

c. West Greenlandic verbalizers: $[\![[_{\text{V}}\ v\ \sqrt{\text{-GAR}}\ ]]\!] =$
write$(\langle v, \sqrt{\text{WÈI}} \rangle)$ >>= $\lambda y.\, \eta [\![v]\!] = \langle [\![v]\!], \{\langle v, \sqrt{\text{-GAR}} \rangle\} \rangle$
(a verbalizer colored by $\sqrt{\text{-GAR}}$ [i.e., 'have', phonologically bound])

Monadic composition for the working linguist

We have seen how monad helps us tease apart logical computation and root coloration in natural language semantics. Now let's see how this technique changes linguists' daily work (I abbreviate $\langle X, \surd \rangle$ as $X_{\surd}$).

Example 1: *wǔ duǒ huā* 'five CLF flower; five flowers' (simplified from Li 2013)

NumP $\langle [\![\text{Num}]\!]([\![\text{Cl}]\!]([\![n]\!])), \{\text{Cl}_{\surd\text{D}}, n_{\surd\text{H}}\}\rangle$ ☜ $([\![\text{N}]\!] >\!\!>\!= \lambda u.\, \eta\langle(\pi_1[\![\text{Cl}_{\surd}]\!])u, \pi_2[\![\text{Cl}_{\surd}]\!]\rangle) >\!\!>\!= \lambda w.\, \eta([\![\text{Num}]\!]w)$

↑ (a plural entity with cardinality 5; it is idiosyncratically characterized by $\surd$HUĀ, and its atomic unit, by $\surd$DUǑ)

- Num 5 $\lambda Q \lambda z.\, {}^*Q(z) \wedge |z|{=}5$ (* is pluralization)
- ClP $\langle [\![\text{Cl}]\!]([\![n]\!]), \{\text{Cl}_{\surd\text{DUǑ}}, n_{\surd\text{HUĀ}}\}\rangle$
  - Cl $\langle [\![\text{Cl}]\!], \{\text{Cl}_{\surd\text{DUǑ}}\}\rangle$
    - Cl $\lambda P \lambda x.\, x \in Atom(P)$ (Cl performs atomization)
    - $\surd$DUǑ
  - N $\langle [\![n]\!], \{n_{\surd\text{HUĀ}}\}\rangle$
    - *n* $\lambda y.\, entity(y)$
    - $\surd$HUĀ

For linguists who don't want such high granularity, the root-level structure can be safely omitted, which would give us just the conventional analysis with no lexical decomposition.

We have seen how monad helps us tease apart logical computation and root coloration in natural language semantics. Now let's see how this technique changes linguists' daily work (I abbreviate $\langle X, \sqrt{}\rangle$ as $X_{\sqrt{}}$).

Example 2: *Mary walks* (based on Bowers 2010, via Lohndal 2019)

VoiceP $\langle ([\![\text{Voice}]\!]([\![v]\!]))[\![\text{Mary}]\!], \{v_{\sqrt{\text{WALK}}}\}\rangle$ ☜ $(([\![\text{V}]\!] \ggg\!= \lambda z.\eta([\![\text{Voice}]\!]z)) \ggg\!= \lambda w.\eta(w([\![\text{Mary}]\!])))$

↑ (an event whose Agent is Mary; it is idiosyncratically characterized by $\sqrt{\text{WALK}}$)

DP Mary

VoiceP $\langle [\![\text{Voice}]\!]([\![v]\!]), \{v_{\sqrt{\text{WALK}}}\}\rangle$

Voice $\lambda P \lambda y \lambda x.\ [P(x) \wedge Agent(x,y)]$

(Voice introduces Agent)

V $\langle [\![v]\!], \{v_{\sqrt{\text{WALK}}}\}\rangle$

v $\lambda x.\ event(x)$

$\sqrt{\text{WALK}}$

Due to the upward continuation of monadic computation, the current theory has global ramifications for semantic composition.

Overview

Category-theoretic setting

Although the application of monad in this work is built on that in Asudeh & Giorgolo (2020), the category-theoretic settings in the two works are different. This is due to the different linguistic frameworks being adopted:

- A&G: categorial grammar (with glue semantics) $\Rrightarrow$ Lambekian
- This work: transformational generative grammar (with ordinary formal semantics [the Heim & Kratzer system]) $\Rrightarrow$ Chomskyan

Cartesian closed category

The category-theoretic context of our monad is a cartesian closed category $\mathbf{Syn}$ of semantics types (as our working metalanguage is simply typed λ-calculus), which can be viewed as a tiny subcategory of the category $\mathbf{Set}$ of sets.

Remark: Since in Chomskyan linguistics

1. linear word order is beyond the purview of the syntax module, and
2. terms are quite freely "copied" in transformational rules,

we don't need to worry about left/right directionality (unlike in Lambek calculus) or resource interpretation (i.e., no need of linear logic).

Category-theoretic setting

The categorical environment we use is quite basic, but it does provide all we need to semantically model Root Syntax.

Monad (adapted from Asudeh & Giorgolo 2020)

We define a monad $\langle T, \eta, \mu \rangle$ s.t. for any $\mathbf{Syn}$-object A, $TA := \langle A, \{\langle X, \surd_1 \rangle, \langle Y, \surd_2 \rangle, \ldots\} \rangle$, where $\langle X, \surd_1 \rangle$ etc. are the linguistic category-root pairs occurring in A. For any $\mathbf{Syn}$-arrow f, $Tf := \lambda \langle x, Q \rangle . \langle f(x), Q \rangle$, where Q is a set of linguistic category-root pairs.

Let H and R be the set of linguistic categories and that of roots in a language, then a set of linguistic category-root pairs is a member of the power set $\mathcal{P}(H \times R)$, and the endofunctor $T := - \times \mathcal{P}(H \times R)$ sends each semantic type to a product type.

The natural transformations η and μ are defined as

- $\eta_A x := \langle x, \emptyset \rangle$ (embedding $x \in A$ in a trivial monadic context TA)
- $\mu_A(\langle \langle x, P \rangle, Q \rangle) := \langle x, P \cup Q \rangle$ (merging two sets of linguistic category-root pairs)

With μ, we can further define >>= on a term ta of type TA and a function f of type $A \to TB$ as ta>>=$f := \mu_B(Tf(ta))$, which yields a term tb of type TB.

Categorical linguistics

Root Syntax spirit

The separate treatment and systematic unification of compositional and idiosyncratic information in theoretical linguistics

This is reminiscent of Coecke et al.'s (2010 et seq.) treatment of compositional and lexical meanings in their Compositional Distributional Semantics (aka DisCoCat) for NLP, though the two lines of work are otherwise highly different (in both linguistic assumptions and research goals). They model natural language syntax (expressed in categorial grammar) in terms of type-tagged vector spaces:

(13) a. $(V \otimes S \otimes W, n^r s n^l)$ (vector space semantics for vt. in DisCoCat)

b. $\langle [\![\mathrm{V}]\!], \{\langle \mathrm{V}, \surd \rangle\} \rangle$ (monadic semantics for bare verbs in Root Syntax)

Coecke et al.'s meaning space largely corresponds to our root log space, and their type tag corresponds to our logical form. This parallelism transcends huge disciplinary differences and might be worth further investigation.

Thank you!

Selected references I

Asudeh, A. & G. Giorgolo
Enriched meanings
OUP, 2020

Borer, H.
Structuring sense: Taking form (Vol. 3)
OUP, 2013

Coecke, B., M. Sadrzadeh & S. Clark
Mathematical foundations for a compositional distributional model of meaning
2010, arXiv:1003.4394

Heim, I. & A. Kratzer
Semantics in generative grammar
Blackwell, 1998

Selected references II

Marantz, A.
No escape from syntax
UPenn working papers in linguistics 2, Vol. 4 201–225, 1997

Song, C.
On the formal flexibility of syntactic categories
University of Cambridge dissertation, 2019

Song, C.
A typology of semilexicality and the locus of grammatical variation (https://youtu.be/2X0-LODELfc)
Talk at ICFL9, 2021a

Song, C.
On the semantics of root syntax (https://youtu.be/1XBCldhL9HQ)
Talk at LENLS18, 2021b