Grammatical types that cannot be: A category-theoretic theorem for Chomsky-school generative syntax*

Extended abstract for ACT2020 (v2.0)

Chenchen (Julio) Song

1 Background

The potential role of category theory as a universal language of science and philosophy is evidenced by its wide interdisciplinary application. It has also been successfully applied to linguistics (e.g., Lambek 1988 et seq., Dougherty 1993, Moortgat 2007, 2009, Gilliber & Retore 2014, Preller 2014, Piedeleu et al. 2015). However, so far the linguistic application of category theory has been limited to two subfields: (i) categorial grammar and its offshoots (based on Lambek calculus); (ii) natural language processing. Notable exceptions are Pollard's (2004 et seq.) higher-order grammar, which is an extension of HPSG, and Lecomte's (2005, 2008) categorial minimalist grammar (see also Amblard, Lecomte & Retoré 2010 and Amblard 2011), which combines categorial and minimalist (i.e., Chomskyan) grammatical theories. However, since both approaches are about the algebraic computation of sentences and essentially rely on Lambek's theory, it is fair to say that they are still within the above-mentioned subfields.

Results in the two subfields above are no doubt exciting, but they can hardly represent the full spectrum of interests in linguistics. In particular, they more or less all focus on the logical-computational side of language. By comparison, working linguists are also (and in fact often more) interested in the typological and analytical aspects of language. Against this background, the current paper presents a linguistic application of category theory outside categorial grammar and NLP. By putting forward a theorem for Chomsky-school syntax, I show that theoretical linguistics may provide rich ground for experimenting category-theoretic ideas as well.

*This is a slightly revised version of my original abstract. Special thanks to the anonymous reviewers of ACT2020 who gave me encouraging and constructive feedback. Please let me know at cs791@cantab.ac.uk if you have any comments/questions regarding this study.

2 Minimalism and phase-based derivation

While previous applications of category theory in linguistics are mostly implemented in categorial grammar, the application reported in this paper is done in the Chomskyan framework—more exactly in *phase-based minimalism*. Minimalism is the latest version of Chomsky's transformational-generative grammar (Chomsky 1995), and phase theory is a crucial recent addition to the minimalist toolkit (Chomsky 2001). Minimalism assumes the architecture in Figure 1 for the human language faculty, where linguistic information flows from the mental lexicon via the syntax to the two interpretative interfaces.

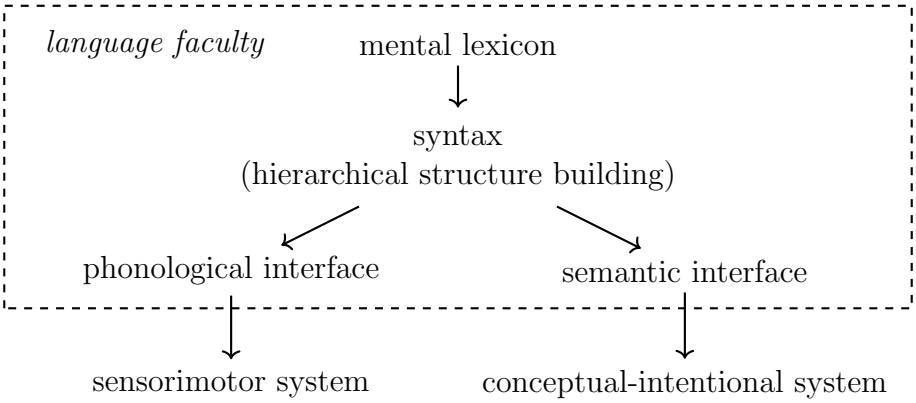


Figure 1: information flow in the human language faculty

The arrows in Figure 1 indicate how sentences are generated from basic units. For example, to generate the sentence *Dan eats cookies*, the lexicon provides an array of words {Dan, eat, cookies, v, T, C...} (v, T, etc. are functional "keywords"), the syntax maps the array to a hierarchically structured object, say [C [$T_{/\text{-s/}}$ [Dan [v [eat cookies]]]]], and the two interfaces map this object to a phonological and a logical form, which in turn are mapped to sensorimotor signals and conceptual-intentional messages.

The bulk of research in minimalism focuses on how the pivotal syntax module works. Phase theory says that the syntax-to-interface information flow in a sentence-generation procedure (i.e., a derivation) happens in multiple cycles or "phases," each consuming a subpart of the lexical array. See the flowchart in Figure 2 for an illustration. Since breaking down structure building into small chunks reduces cognitive-computational burden, phase-based derivation is deemed by generative syntacticians as highly economical.

A major task of phase theory is to identify "phase heads" (a head is the core unit of a phrase), which serve to define lexical subarrays and phasal zones. The overall number of phase heads in natural languages is a matter of ongoing research.

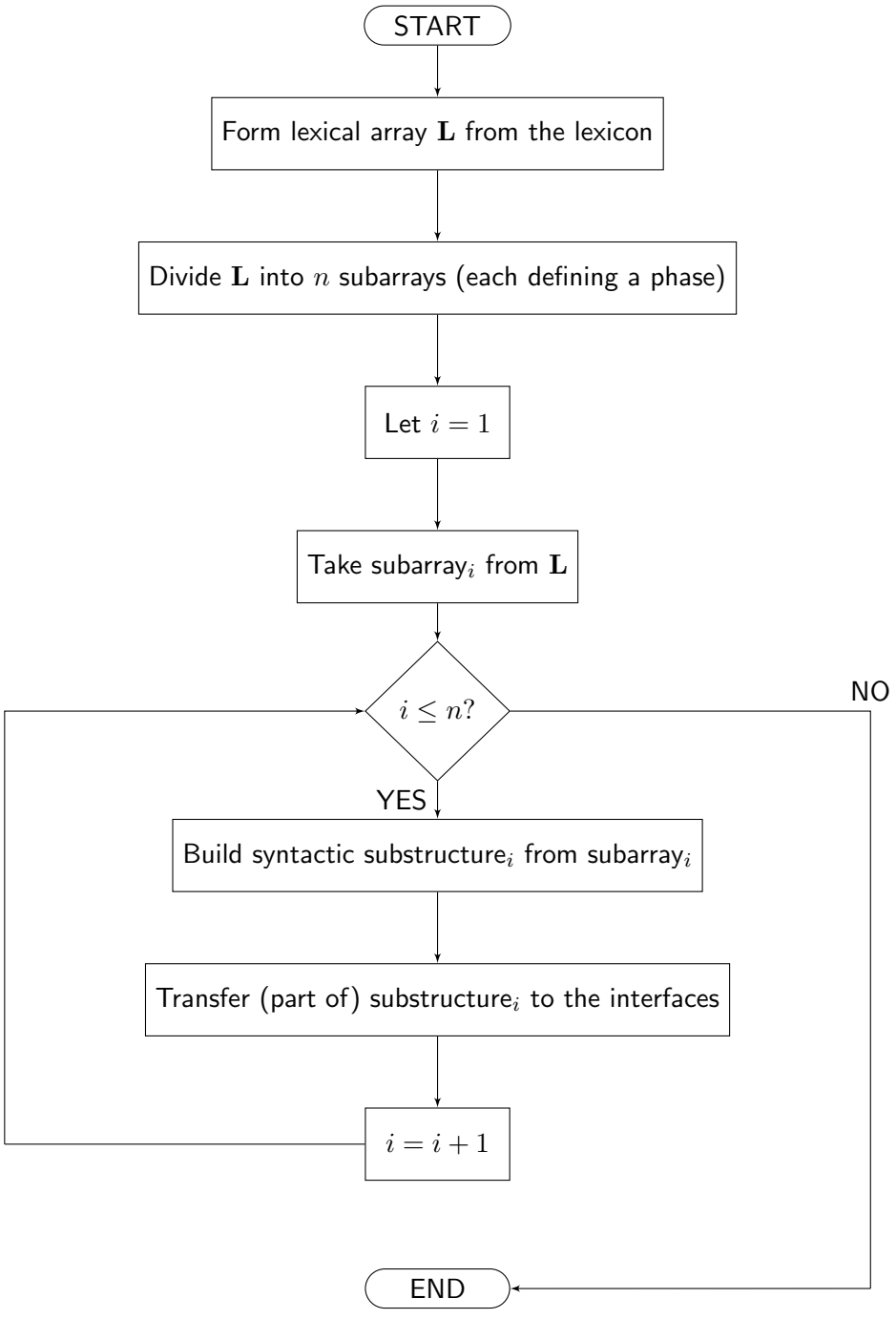


Figure 2: phase-based derivation in minimalism

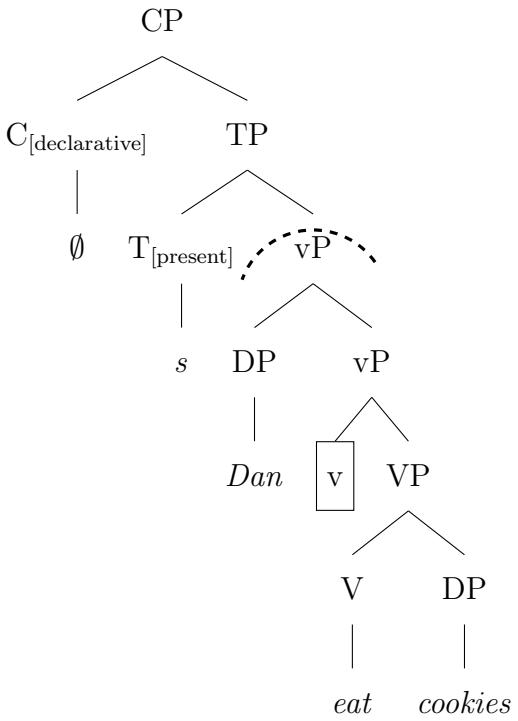
3 Little x phase heads and their disputed "flavors"

Among the phase heads that linguists have proposed, there is a somewhat special group usually denoted by lowercase x (colloquially "little x"), where x stands for a *major part of speech*, such as v for verb and n for noun. The little x head demarcates a complete core semantic (aka thematic) zone in the derivation. In the case of v it signals a completely built eventuality (before further semantic layers like tense and modality are added). To illustrate, the eventuality in *Dan eats cookies* is a simple event of eating. By adding thematic predicates to this eventuality we obtain the verb's argument structure, as in (1).

(1) EAT(e) & AGENT(e, Dan) & THEME(e, cookies)

The syntactic reflection of (1) is built up in the v phase—the maximal phrase headed by v (i.e., the biggest vP)—as illustrated in the tree diagram in (2). The dashed arc marks the phase boundary.

(2)



A phase consists of three structural parts: the phase head (here the boxed v), the phase domain (here the VP subtree), and the phase edge (here the [DP [*Dan*]] subtree). These parts respond differently to the transfer-to-interface operation, but that is outside the scope of this paper. What makes little x heads—in particular little v—special is that multiple semantic "flavors" have been proposed for them, such as v_{DO}, v_{BE}, and v_{CAUSE}. The little v flavors are mainly used to account for event type–related phenomena. Thus, the v in *Dan eats cookies* is v_{DO} (for eating is an activity), while the v's in the two Spanish

sentences in (3) are respectively v_{BE} (for lacking something is a state) and v_{GO} (for arriving somewhere is a change of location).

(3) a. [$_{v_{GO}}$ *Llegaron*] *dos cartas.* [Spanish]
arrived.GO two letters

'Two letters arrived.'

b. [$_{v_{BE}}$ *Faltan*] *dos velitas en la torta.*
lack.BE two little candles in the cake

'Two birthday candles are missing from the cake.' (Cuervo 2003)

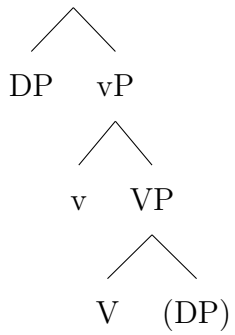
Similarly, the v heads associated with the two Japanese verbs in (4) are also different: one is v_{BECOME} (for getting up is a chance of state) and other is v_{CAUSE} (for waking someone up is a causative event).

(4) a. *okiru* 'get up (v_{BECOME})' [Japanese]

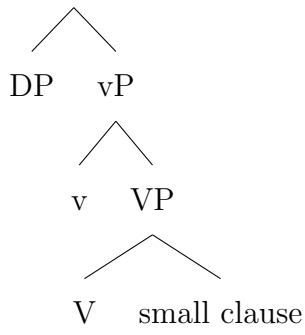
b. *okosu* 'wake someone up (v_{CAUSE})' (Harley 2008)

The flavor-oriented thinking is very popular, especially among syntacticians influenced by the distributed morphology theory. That being said, quite a few researchers have raised doubts about the theoretical status of flavored heads (see, e.g., Acedo-Matellán & Mateu 2014 and Alexiadou & Lohndal 2017). A major motivation behind such doubts is that flavor-based analyses of the relevant linguistic phenomena are not irreplaceable. Thus, Acedo-Matellán & Mateu replace the flavor-based account of event type variation with a configuration-based account, based on which we can replace v_{DO} with the configuration in (5a) and v_{CAUSE} with that in (5b).

(5) a.



b.



To illustrate, (6a) and (6b) have (5a) as (part of) their underlying structure, while (6c) has (5b). All three sentences assume the same, unflavored v head, and no aspects of their event/argument structures cannot be explained by the configurations per se.

(6) a. Dan [$_{vP}$ v [$_{VP}$ [$_{V}$ eats]]].

b. Dan [$_{vP}$ v [$_{VP}$ [$_{V}$ eats] [$_{DP}$ cookies]]].

c. Dan [$_{vP}$ v [$_{VP}$ [$_{V}$ eats] [$_{\text{small clause}}$ the cookies up]]].

From a metatheoretical perspective, *ceteris paribus*, a flavor-free analysis of a certain phenomenon is superior to a flavor-based one, because the former postulates fewer functional keywords than the latter and hence is more parsimonious. There are at least two other reasons to question the legitimacy of flavored little x heads:

1. No other phase heads except little x's have flavors. Nor do the flavors of little x's have any phase-theoretic significance.

2. The postulation of little x flavors requires extra formal syntactic features (to define the flavors), but it is unclear how children can successfully postulate them in the process of language acquisition.

In particular, point 2 reflects what Biberauer & Roberts (2015, 2017) call "feature economy," which they define as a general cognitive bias behind language acquisition (see also Biberauer 2017).

4 Interim summary

In the current version of transformational-generative grammar (i.e., minimalism) syntactic derivation proceeds in multiple cycles called *phases*. Phase theory is a most important new component in post-2000 Chomsky-school syntactic research and is still under active development. Phases are defined by *phase heads*. Among the phase heads that have been postulated, the little x's appear to be special, as they come in different semantic flavors, such as DO, BE, and CAUSE for v. However, the theoretical status of little x flavors is disputed. There are three main arguments against them:

1. They are not necessary since the relevant phenomena can have alternative analyses.

2. They make little x's unlike all other phase heads without a clear reason why.

3. They rely on extra formal syntactic features but researchers who use them do not engage with the issue of how those features can be naturally postulated by children.

Despite the high intuitive desirability to eliminate flavored little x's, these existing arguments are not strong enough to really rule them out. First, that the relevant phenomena can have alternative analyses is no proof that the flavor-based explanation is wrong. Second, that previous studies do not explain why only little x's but not other phase heads have flavors does not mean no such explanation can be found. Third, that

researchers who use flavored little x's have not engaged with the acquisition issue properly does not mean the issue is an insurmountable obstacle for them. In Section 5 I will apply category theory to Chomskyan syntax and show that little x flavors must be eliminated if we want to maintain an ontological tenet in minimalism about syntactic heads.

5 Category theory rules out flavored phase heads

Syntactic heads (e.g., V, T, C) are ground terms in the Chomskyan syntactic derivation system. Each head corresponds to an atomic (aka minimal) syntactic category. To avoid confusion I will refer to syntactic categories as *grammatical types* hereafter and reserve the term "category" for mathematical categories. The inventory of grammatical types has rich internal structures, which can be given a category-theoretic modeling.

Current generative syntax employs a rather large type inventory far beyond the familiar parts of speech in school grammar. Which languages make use of what types is a point of considerable variation, and researchers may also omit certain heads in their analyses when those are irrelevant to the problem at hand. In a word, despite the large number of ground types in current generative syntax, their actual usage is quite flexible.

All grammatical types in the same major part of speech fall in a combinatorial scope–based ontological sequence commonly called an "extended projection" or "hierarchy of projection." But we can do without these theory-internal terms, because such sequences are simply *partially ordered sets*. The combinatorial scope of a grammatical type is just its relative height on the syntactic tree. Take the tree in (2) on page 4 for example. The "spine" of the tree consists of four heads, whose scope-based ordering is given in (7).

(7) $\mathrm{V} \leq \mathrm{v} \leq \mathrm{T} \leq \mathrm{C}$

Flavored heads, being subtle semantic variants (i.e., subtypes) of the same grammatical type and competing for the same tree position, can be modeled by *incomparable elements* in the poset. So, if there are three v flavors v_{DO}, $\mathrm{v}_{\mathrm{BECOME}}$, and $\mathrm{v}_{\mathrm{CAUSE}}$ in (7), then the poset changes to (8).

(8) $\mathrm{V} \leq \begin{matrix} \mathrm{v}_{\mathrm{DO}} \\ \mathrm{v}_{\mathrm{BECOME}} \\ \mathrm{v}_{\mathrm{CAUSE}} \end{matrix} \leq \mathrm{T} \leq \mathrm{C}$

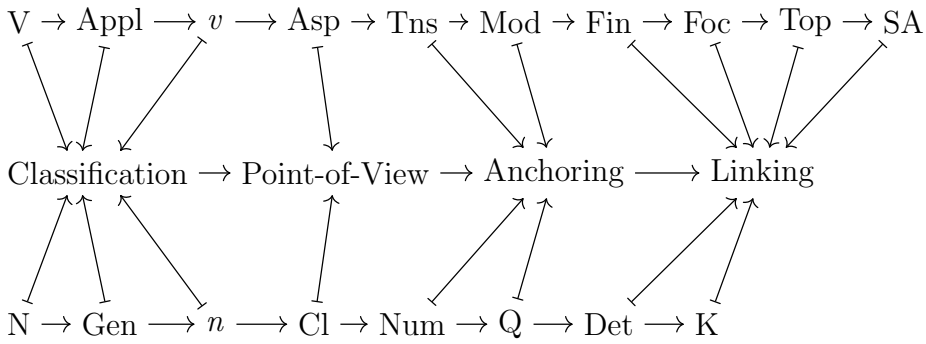
Partial orders are not the only structure in the grammatical type inventory. Generative syntacticians also talk about a conceptual parallelism between extended projections of different major parts of speech (e.g., tense is to verb what determiner is to noun). This idea has been developed into a *universal spine hypothesis* (Wiltschko 2014), which postulates a shared conceptual template for extended projections. I present the template in (9) in a partially ordered format. It is obviously a chain.

(9) classification $\leq$ point-of-view $\leq$ anchoring $\leq$ linking

The abstract concepts in (9) correspond to complete functional zones (i.e., subsets) in extended projections. For example, classification corresponds to the core semantic zone; namely, the zone where the classification of events or individuals is completed. Crucially, phases also correspond to complete functional zones. Admittedly, perhaps not every Wiltschkovian zone is a phase zone (though this is still an open issue). But insofar as current phase theory is concerned, phase heads (e.g., *v*, C, D) always occur at functional zone–boundary positions. More exactly, they always occur at the upper boundary of functional zones. For example, *v* is at the upper boundary of the classification zone, and C is at the upper boundary of the anchoring zone. This is an expected situation because phase heads are meant to trigger the cyclic spell-out of derivational subparts that are relatively complete in interface (i.e., semantic and phonological) interpretation, which means that they can only appear at positions where such relatively complete zones have been completely built.

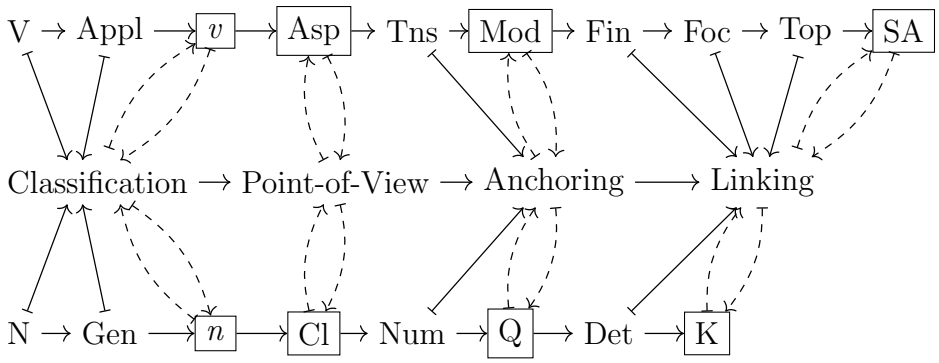
Despite the much interest it has given rise to, the cross-major-part-of-speech parallelism has not been given a formal definition, partly because it is hard to be concretely pinned down. For one thing, the parallelism is far from a one-to-one correspondence; for another, except for a few *core grammatical types* (such as v and T), most other types do not have suitable cross-major-part-of-speech counterparts. The universal spine hypothesis offers a potential solution to this problem, since it opens the gate to a higher-order correspondence across extended projections—that is, a correspondence between entire zones of grammatical types instead of the individual types themselves. I illustrate this mediated correspondence in (10) with two longer extended projections.

(10)



Since all three rows in (10) are mathematically posets, the mappings between them can be viewed as monotone functions (the monotonicity is easy to check). Moreover, we can add another pair of linguistically meaningful monotone functions to (10)—by mapping elements in the universal spine to zone boundary–marking grammatical types in the two extended projections. I illustrate this in (11), highlighting the boundary-marking types with boxes and the invertible mapping instances with dashed arrows.

(11)



From a category-theoretic perspective, (11) has two pairs of functors between three posets viewed as categories—call them $\mathcal{A}_{\mathbf{V}}$ (the verbal poset), $\mathcal{A}_{\mathbf{N}}$ (the nominal poset), and $\mathcal{A}_0$ (the universal spine chain). What is more, each pair of back-and-forth functors actually form an *adjunction*. I present the adjunction in (12).

(12) $\mathcal{A}_{\mathbf{V}} \underset{e_{\mathbf{V}}}{\overset{h_{\mathbf{V}}}{\perp}} \mathcal{A}_0 \underset{e_{\mathbf{N}}}{\overset{h_{\mathbf{N}}}{\perp}} \mathcal{A}_{\mathbf{N}}$

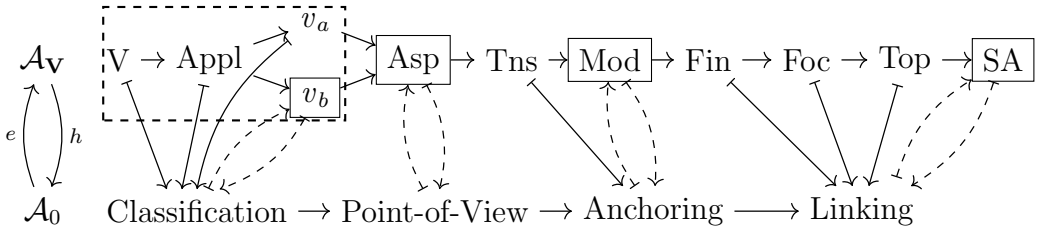
Two categories that are connected by functors may be similar in three senses: that they are isomorphic, equivalent, or connected by an adjunction. In the case of the cross-major-part-of-speech parallelism, isomorphism can be ruled out right away, for $\mathcal{A}_{\mathbf{V}}$ and $\mathcal{A}_{\mathbf{N}}$ as posets have different cardinalities, which means it is impossible to construct a bijection between them. Next, equivalence of categories is ruled out as well, since two poset categories are isomorphic iff they are equivalent. Thus, if a category-theoretic similarity between the major parts of speech can be constructed at all, it must be an adjunction—and more specifically, the above universal spine–mediated adjunction is the only linguistically meaningful possibility.

Before continuing my category-theoretic discussion, I want to make a comment on the issue of linguistic meaningfulness, which is a critical criterion in judging whether a certain mathematical choice is suitable for our linguistic task. From a purely mathematical perspective, (11) is not the only way to construct an adjunction between an extended projection and the universal spine—we could also choose to map the Wiltschkovian concepts to grammatical types at the lower boundary of each functional zone. However, that choice is counterintuitive in the context of the functional zone–oriented thinking; in other words, it is not a linguistically meaningful choice. Specifically, no matter which functional zone template we choose to work with, be it the universal spine I am adopting here or the equally popular template "thematic–inflectional–discourse–speech act," the linguistic significance and motivation of such partitioning lies in the idea that it defines *functionally complete* subderivations (i.e., subtrees). As such, if we are to choose a "representative" for each functional zone (whatever its label) among all the grammatical types it subsumes—to the effect that whenever that type is mentioned, it would immediately remind us of a

functionally complete derivation zone—the only suitable choice is the type that lies at the end of the zone,[1] but not the type that lies at its beginning or some random middle point. The alternative choices are mathematically valid but deviant from the whole purpose of the functional zone–oriented thinking. Hence, they are linguistically uninteresting.

With the above category-theoretic setting laid out and choices of adjoint functors motivated, we can now turn to consider the influence of flavored grammatical types on the mediated adjunction between major parts of speech. Take the flavored little v's for example, as in (13).

(13)



As we can see, when v is flavored, only one of its flavors (say v_b) can be hit by the functor e, for monotone functions are single-valued. But this situation runs afoul of the *adjoint functor theorem* in category theory. Following is the poset version of the theorem.

Adjoint functor theorem for posets. *Suppose the two posets $\mathcal{P}_{\leq}$ and $\mathcal{Q}_{\sqsubseteq}$ viewed as categories are in an adjunction, and suppose $\mathcal{P}_{\leq}$ has all joins. Given a monotone function $f\colon \mathcal{P}_{\leq} \to \mathcal{Q}_{\sqsubseteq}$ viewed as a functor, its right adjoint $g\colon \mathcal{Q}_{\sqsubseteq} \to \mathcal{P}_{\leq}$ is uniquely determined by the following formula for any $q \in \mathcal{Q}_{\sqsubseteq}$:*

$$g(q) = max\{p \in \mathcal{P}_{\leq} \mid f(p) \sqsubseteq q\}$$

Dually, if $\mathcal{Q}_{\sqsubseteq}$ has all meets, then given a functor $g\colon \mathcal{Q}_{\sqsubseteq} \to \mathcal{P}_{\leq}$, its left adjoint $f\colon \mathcal{P}_{\leq} \to \mathcal{Q}_{\sqsubseteq}$ is uniquely determined by the following formula for any $p \in \mathcal{P}_{\leq}$:

$$f(p) = min\{q \in \mathcal{Q}_{\sqsubseteq} \mid p \leq g(q)\}$$

Example (13) does not satisfy the conditions of the adjoint functor theorem because the "classification zone" subposet in the verbal extended projection (as indicated by the dashed box) has no maximum. That is, there is no such element in $\mathcal{A}_{\mathbf{V}}$ that is both an upper bound of the boxed subposet and part of it. And this problem is not unique to flavored little v's—it occurs to any flavored grammatical types at the linguistically meaningful zone-boundary positions (here v, Asp, Mod, and SA but the concrete types may vary depending on the particular set of types one is working with). As such, (13) does not qualify as an adjoint situation.

[1] This is also a standard choice in the field, as evidenced by widely used terms like the "vP-domain," which refers to the derivational domain culminating with vP but not that beginning with it.

I take the failure of (13) to satisfy the adjoint functor theorem to mean that if we want to maintain the conceptual parallelism across major parts of speech at all, then no upper-boundary grammatical types can be flavored (NB this does not preclude lower-boundary or non-boundary types from being flavored). In particular, this means that no phase heads can be flavored, because phase heads always occur at upper boundaries of functional zones. This further conclusion is the most important result in this paper, so I present it as a theorem.

Theorem 1 (uniqueness of phase heads)**.** *In phase-based minimalism, no phase head can be flavored.*

Thus, category theory provides an additional and much stronger motivation for the elimination of flavored little x's from generative syntax.

6 Conclusion

In this paper I have applied category theory to model the inventory of grammatical types in current Chomsky-school generative grammar (i.e., minimalism with phases). Under this modeling, I have reached the conclusion that flavored phase heads, in particular flavored little v's, should be eliminated from the minimalist program. The structure of my argument is as follows:

1. From a category-theoretic perspective, the grammatical types of a language are organized in a particular subcategory of the category of posets. These posets are called extended projections and are hierarchies of types belonging to the same major part of speech.

2. Under the above category-theoretic perspective, in order to maintain the cross-part-of-speech parallelism (which is a basic observation in generative syntax) we need to pin down some sort of similarity between extended projections qua categories.

3. After ruling out an isomorphism and an equivalence of categories, we are only left with the possibility of an adjoint situation (which defines a weak similarity in line with the linguistic observation). And the only linguistically meaningful adjoint situation we can get is an indirect one mediated by a functional zone–based conceptual template (the particular version of the template I have adopted is Wiltschko's "universal spine"). The configuration involves two pairs of adjoint functors, each between an extended projection and the conceptual template.

4. Moreover, the only linguistically meaningful adjunction between any extended projection and the conceptual template is one where the particular functor h from the former to the latter has a right adjoint, which maps each abstract concept in the

template to the grammatical type lying at the upper boundary of its functional zone. Since our categories are posets, under the adjoint functor theorem this amounts to saying that the functor h preserves all joins, when joins exist in the extended projection poset.

5. In a variant of minimalist syntax with "flavored" little x phase heads, those extended projections where the little x is flavored do not have join-preserving functors to the universal spine template. Hence, if we accept 1–4 above we have to dispense with flavored little x's.

Since phase theory plays an important role in current Chomskyan syntax, the formulation of such a mathematical theorem is highly valuable. Finally, beyond the immediate goal of this paper, the category-theoretic modeling of the Chomskyan grammatical type system may still offer more linguistically interesting results. This suggests that the application of category theory to linguistics need not be limited to computationally oriented subfields. Theoretical linguistics per se can also benefit from this powerful mathematical tool.

Selected references

[1] Chomsky, Noam (1995). *The Minimalist Program.* MIT Press, Cambridge MA.

[2] Chomsky, Noam (2001). Derivation by phase. In Kenstowicz, M. (ed.), *Ken Hale: A Life in Language*, pp.1–52. Oxford University Press, Oxford.

[3] Biberauer, Theresa (2017). Factors 2 and 3: A principled approach. In Song, C. & J. Baker (eds.), *Cambridge Occasional Papers in Linguistics*, vol. 10, pp.38–65.

[4] Wiltschko, Martina (2014). *The Universal Structure of Categories: Towards a Formal Typology.* Cambridge University Press, Cambridge.